Appendix 2. The Maxwell-Hertz equations and the dynamic field equations in the mobile linear and nonlinear medium

1. In introduction we have found out, that the method of a relaxation is based on algorithm of calculations of full derivatives on time from **E** and **H** in the left parts of the equations (9) and (10). This difference from Maxwell equations was not essential while it was a question of the immobile mediums. Further it is impossible already to miss from attention. Besides, if we shall pass now to a solution of a nonlinear problem we should consider, that in nonlinear medium the motion of local increases ε or μ after field maxims demands modification and in field equations. Thus, the medium nonlinearity compels us to fall outside the limits the standard static statement of a problem. Below we shall show it.

First usual Maxwell equations we shall lead to a view (9) and (10), having allocated full derivatives on a time from inductions. The purpose will be reached if to take advantage of the equality, which is fair for any vectors and their velocity **U** (see, for example [7]):

$$\frac{d}{dt}\mathbf{D} = \frac{\partial}{\partial t}\mathbf{D} + \frac{d\mathbf{D}}{d\mathbf{U}} \quad \text{and} \quad \frac{d}{dt}\mathbf{B} = \frac{\partial}{\partial t}\mathbf{B} + \frac{d\mathbf{B}}{d\mathbf{U}}, \qquad (\Pi 2.1)$$

where $\mathbf{D} = \varepsilon_0 \varepsilon_e \mathbf{E}$ and $\mathbf{B} = \mu_0 \mu_e \mathbf{H}$.

Let's add a corresponding derivative on velocity $\frac{d\mathbf{D}}{d\mathbf{U}}$ or $\frac{d\mathbf{B}}{d\mathbf{U}}$ to the left and right part of each of Maxwell equations of a view (3) and (4) (see Introduction). Then we shall gain such equations of the field in moving medium:

$$\frac{d}{dt}\mathbf{D} = rot\mathbf{H} + \frac{d\mathbf{D}}{d\mathbf{U}}. \qquad (\Pi 2.2)$$

$$\frac{d}{dt}\mathbf{B} = -rot\mathbf{E} + \frac{d\mathbf{B}}{d\mathbf{U}}. \qquad (\Pi 2.3)$$

Formulas (П2.2) and (П2.3) are nothing more nor less that only a little bit the changed the notation shape of Maxwell equations in which velocity has been added after simple transformations in an obvious view.

Now we shall make two basic additions to equality (П2.2) and (П2.3). First, when we speak "velocities", we mean velocities of equivalent dielectrics, which are connected, with maximums of fields through nonlinear dependences. The general character of a motion of these maximums at turns to us is already known from researches of waves in the closed linear wave-guides. From them directly follows, that these two velocities take place at rotation of electromagnetic fields, instead of one, as, for example, as in Hertz

[18]. The motion of a maximum of an electric field is connected with one of them, and with another - a motion of a magnetic field. If velocity was a single, these maximums could not converge and became parted during a mode changing of a standing wave to a mode of a running wave and back. Therefore we should put the separate velocity in each of formulas (П2.2) and (П2.3). Secondly, we use following equality, which follow from a rule of differentiation:

$$\frac{d}{dt}\mathbf{D} = \varepsilon_0\varepsilon\frac{d\mathbf{E}}{dt} + \varepsilon_0\mathbf{E}\frac{d\varepsilon}{dt} \quad \text{and} \quad \frac{d}{dt}\mathbf{B} = \mu_0\mu\frac{d\mathbf{H}}{dt} + \mu_0\mathbf{H}\frac{d\mu}{dt} \ . \qquad (\Pi 2.4)$$

They are necessary for the equations transformation to a view convenient for the computing. As a result we shall gain the equations of the field in a following view:

$$\frac{d\mathbf{E}}{dt} = \varepsilon^{-1}\left(\frac{1}{\varepsilon_0}rot\mathbf{H} + \frac{d(\varepsilon\cdot\mathbf{E})}{d\mathbf{U}e} - \mathbf{E}\frac{d\varepsilon}{dt}\right), \qquad (\Pi 2.5)$$

$$\frac{d\mathbf{H}}{dt} = \mu^{-1}\left(-\frac{1}{\mu_0}rot\mathbf{E} + \frac{d(\mu\cdot\mathbf{H})}{d\mathbf{U}h} - \mathbf{H}\frac{d\mu}{dt}\right), \qquad (\Pi 2.6)$$

where $\mathbf{U}e$ and $\mathbf{U}h$ are velocities of electrical and magnetic fields.

Expressions (П2.5) and (П2.6) represent the Maxwell equations for a moving medium with variable parameters ε and μ.

2. It is necessary to note, that the Maxwell did not give a lot of attention especially to the mobile mediums. Such approach has been justified by that studying of properties of an electromagnetic field at that time only began. It has allowed reducing all 20 equations from 20-th unknown to two a vector equality of a view (3), (4). Henry Hertz has made the attempt to inject a velocity into field equations of [18], considering process at an elementary level. Its result has not proved to be true at that time experiment. However now any attempts of solutions in this direction represent for us exclusive interest. Therefore we are compelled to return to its solution again. We shall consider it.

If to apply vector algebra, Hertz equations for medium without currents can be written down in a compact view (see, for example, [19]) so:

$$\begin{aligned} &\frac{\partial}{\partial t}\mathbf{D} + rot[\mathbf{DU}] + \mathbf{U}\cdot div\mathbf{D} = rot\mathbf{H}, \\ &\frac{\partial}{\partial t}\mathbf{B} + rot[\mathbf{BU}] + \mathbf{U}\cdot div\mathbf{B} = -rot\mathbf{E}. \end{aligned} \qquad (\Pi 2.7)$$

Expressions (П2.11) and (П2.12) represent Hertz equations for the mobile nonlinear medium. These equations contain in the right part spatial derivative of velocities that essentially distinguishes them from Maxwell equations.

Above we have formulated Maxwell and Hertz equations for a moving medium. However, it is not enough of it for an exhaustive solution of our problem. Nonlinearity in conditions of a curvilinear motion causes changes in representation of performances of medium, and also in the field equations.

3. The question on parameters of medium is closely connected with velocity modes in a vortex. They are closely connected with fundamental properties of the electromagnetic waves propagation. As is known (see, for example, [20]), velocity of distribution of an electromagnetic field has one value only in a homogeneous flat transverse TEM-wave. Also it is known, that flat waves in the nature never happens. Even artificial laser radiation does not possess such properties [21]. Thus, we should reckon with that the vortex consists of longitudinal waves of E-type and H-type, including mixed. The presence of two velocities is always an inherent property of these waves. One of them is a phase velocity - $\mathbf{U}_f$, and the second is a group velocity - $\mathbf{U}_g$. But velocities are jointed by a formula:

$$\mathbf{U}_g \cdot \mathbf{U}_f = c^2, \qquad (\Pi 2.13)$$

where $c = (\varepsilon_0 \varepsilon \cdot \mu_0 \mu)^{-1/2}$ is the velocity of a flat wave, which is not existing in the nature.

The phase velocity $\mathbf{U}_f$ characterizes a velocity of a change of the field phase, i.e. corresponds to a velocity of vectors **E** or **H**. In the equations (П2.5), (П2.6) and (П2.11), (П2.12) we mean motion velocities of an equivalent dielectrics and paramagnetics. These are anomalies of ε and μ. Moving of dielectrics and of paramagnetics is equivalent to an energy carrying. Velocity of this carrying is unequivocally equal to group velocity - $\mathbf{U}_g$. Inductions **D** and **B** are functions as **E**, **H**, and ε, μ. But, as velocities of fields and anomalies ($\mathbf{U}_g$ and $\mathbf{U}_f$) are not equal among themselves motions of pairs **E**, **D** and **H**, **B** is not synchronous. Hence, **E** is not collinear with **D** the same as and **H** with **B**. But it can be only then, when ε and μ are essence tensors - $\boldsymbol{\varepsilon}$ and $\boldsymbol{\mu}$. This fact complicates all calculations since connects in one unit a problem of velocities calculation and a problem of calculation of tensors ε and μ.

Thus, we cannot calculate velocity separately, generally speaking. For each of fields - electrical and magnetic - generally it is necessary to search simultaneously at once for 12 numerical arrays, to corresponding 12 functions of a point, which describe the necessary components of unknown velocities and parameters of medium. And all problems, for example, for an electrical component can be shown to a solution of system of the following equations:

$$\begin{cases} \dfrac{d}{dt}\mathbf{E} = \dfrac{\partial}{\partial t}\mathbf{E} + \dfrac{d\mathbf{E}}{d\mathbf{U}e_f}, \\ \dfrac{d}{dt}\boldsymbol{\varepsilon} = \begin{pmatrix} \dfrac{\partial\varepsilon_{11}}{\partial x}Ue_x + \dfrac{\partial\varepsilon_{11}}{\partial y}Ue_y + \dfrac{\partial\varepsilon_{11}}{\partial z}Ue_z & \ldots & \dfrac{\partial\varepsilon_{13}}{\partial x}Ue_x + \dfrac{\partial\varepsilon_{13}}{\partial y}Ue_y + \dfrac{\partial\varepsilon_{13}}{\partial z}Ue_z \\ \ldots\ldots\ldots\ldots & \ldots & \ldots\ldots\ldots\ldots \\ \dfrac{\partial\varepsilon_{31}}{\partial x}Ue_x + \dfrac{\partial\varepsilon_{31}}{\partial y}Ue_y + \dfrac{\partial\varepsilon_{31}}{\partial z}Ue_z & \ldots & \dfrac{\partial\varepsilon_{33}}{\partial x}Ue_x + \dfrac{\partial\varepsilon_{33}}{\partial y}Ue_y + \dfrac{\partial\varepsilon_{33}}{\partial z}Ue_z \end{pmatrix}. \end{cases} \quad (\Pi 2.14)$$

Thus, naturally, two conditions should be considered: $\mathbf{D} = \varepsilon_0 \boldsymbol{\varepsilon} \cdot \mathbf{E}$ and $\mathbf{U}e \cdot \mathbf{U}e_f = c^2$.

And tensor $\boldsymbol{\varepsilon}$ should be normalized to that quantity which is certain by the equations (П1.9) or (П1.10). Here it is a question of that, as $\boldsymbol{\varepsilon}$ and $\boldsymbol{\mu}$ should suppose the following representation:

$$\boldsymbol{\varepsilon} = \varepsilon \cdot \boldsymbol{\varepsilon}_i, \qquad \boldsymbol{\mu} = \mu \cdot \boldsymbol{\mu}_i, \quad (\Pi 2.15)$$

where $\boldsymbol{\varepsilon}_i$ and $\boldsymbol{\mu}_i$ - tensors which are responsible only for turns relative of axes without the change of their amplitudes.

Let's notice, that the computing problem essentially becomes simpler by reduction of unknown number a component of tensors $\boldsymbol{\varepsilon}$ and $\boldsymbol{\mu}$, if to consider the properties of their elements following from rotation (see, for example [10], item 14.10.7).

4. Now time has come for essence finding-out of those compelled changes in the field equations, which are connected with new statement of a problem.

Formulas (П2.11) and (П2.12) contain functions which depend on velocities. This circumstance already in itself deduces formulas essentially outside the essence of initial Maxwell equations. Its all theory is constructed on balance of streams of field lines. But these streams are force attributes. I.e. Maxwell equations contain balance of some forces in the base. But and inertial forces and accelerations are not present anywhere at its equations. And we know, that the section of mechanics constructed only on balance of forces, is static. Therefore and the classical electrodynamics is in essence a static. It is obvious, that we ejects the equations beyond score of a usual static by when have added in them only velocities. But we do not do them more close to dynamics. Therefore our equations, generally speaking, are not full.

Further we shall take advantage of fundamental properties of Maxwell-Hertz equations and we shall apply them as a base for the further development of the theory from a static electrodynamics up to its dynamic shape. Really, the transition from a statics to a dynamics is in general reduced to the account of a balance of all forces and of all forces moments. Naturally this balance should include inertial forces and the moments of inertial forces. And the forces balance and the balance of moments can be considered separately. It follows from a relativity principle of Galilee and from a Konig theorem for kinetic energy of a material particles system [22].

5. We shall begin with balance of forces.

Inertial forces submit to the second law of Newton:

$$\mathbf{F}e^* = m_e \frac{d\mathbf{U}e}{dt}, \quad \mathbf{F}h^* = m_h \frac{d\mathbf{U}h}{dt}, \tag{П2.16}$$

where $\mathbf{F}e^*$ and $\mathbf{F}h^*$ are electrical both magnetic inertial forces, m_e and m_h are corresponding electrical and magnetic mass densities.

Let's pay attention to that inertial forces enter into the common forces balance additive with static forces, which become now or potential forces (without vortexes) or quasi-potential forces (with vortexes) in a sense of the physical interpretation. We apply this a vague term only to designate the some elasticity of all vortical system (not the absolute rigidity usually set by tie connections, and not the ideal elasticity caused by potential forces).

If now from the forces, containing in Maxwell equations and draft copies of Hertz equations, to subtract inertial forces, then it and will lead to the required dynamic equations. Now it is necessary to coordinate dimensionalities, i.e. to find connection between forces and fields. The usual shape of such connection looks like:

$$\mathbf{F}e = \rho_e \mathbf{E}, \qquad \mathbf{F}h = \rho_h \mathbf{H}, \tag{П2.17}$$

where $\mathbf{F}e$ and $\mathbf{F}h$ are electrical and magnetic forces, ρ_e and ρ_h are corresponding unknown densities of the charges distribution in a medium.

Real charges are absent at us on a problem specification. However there are displacement currents, which is possible to compare to a motion of equivalent charges. We shall find these charges in an elementary cube with the cube side - *dx*, *dy*, *dz* which is presented in Fig. П2.1 at the left. The average charge is identically equal to zero by virtue of full symmetry of a physical picture if medium is homogeneous also a cube is in a homogeneous electric field. However, if the field is inhomogeneous, then with conspicuity ρ_e and ρ_h are defined by a charging asymmetry on opposite sides of a cube and, hence, are tensors. Their diagonal elements can be found through capacities of sides concerning an average surface. The scheme of two equivalent capacitors is represented in Fig. П2.1 on the right. Two sides and an average surface form capacitors.

Here q_1, q_2 are charges on external plates of condensers, C_1, C_2 are capacities of capacitors, V_1, V_2 are corresponding voltages. (We shall notice, that it is possible to replace each of capacitors to a group of parallel capacitors to consider non-uniformity of distribution V_1, V_2, however it changes business a little).

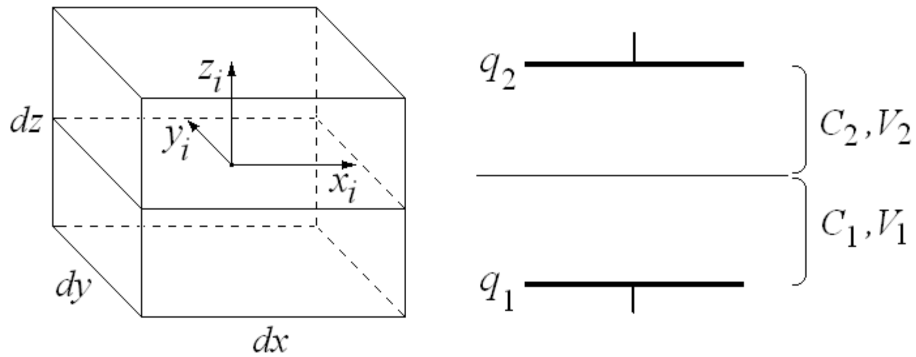

Fig. П2.1

It is possible to write down following formulas on the basis of Fig. П2.1:

$$\Delta q_z = q_2 + q_1 = C_2 V_2 - C_1 V_1 = dxdy \cdot \Delta D_z \ . \qquad (\Pi 2.18)$$

Thus, equivalent charges accompany displacement currents. For example, *z*-th component of the tensor describing distribution of this charge, can be calculated under the formula:

$$\rho_{ezz} = \frac{1}{dxdydz} \Delta q_z = \frac{\partial D_z}{\partial z} \ . \qquad (\Pi 2.19)$$

Other components of tensors $\boldsymbol{\rho}_e$ and $\boldsymbol{\rho}_h$ can be calculated completely similarly. And then, it is possible to calculate by means of them the forces distribution, operating in the mobile field system, under formulas (П2.14).

6. The following variant of use of equalities (П2.13)…(П2.19) opens after the resulted analysis.

First inertial fields $\mathbf{E}^*$ also $\mathbf{H}^*$ are entered into Hertz equations. Then fields can be expressed through inertial forces. From them we find formulas:

$$\mathbf{E}^* = m_e \boldsymbol{\rho}_e^{-1} \frac{d\mathbf{U}e}{dt} \quad \text{and} \quad \mathbf{H}^* = m_h \boldsymbol{\rho}_h^{-1} \frac{d\mathbf{U}h}{dt} \ . \qquad (\Pi 2.20)$$

Here components of tensors $\boldsymbol{\rho}_e, \boldsymbol{\rho}_h$ are defined under formulas (П2.16).

It is necessary to define mass densities of mass m_e, m_h. For this purpose we shall take advantage of Poincare formula (1). But we shall consider, that it is gained formally from Lorentz's transformations. These transformations are constructed for mediums with a certain the constant velocity of interactions propagation. Lorenz has taken as this velocity of a flat wave in the vacuum, which not existing in the nature. However, we assume, that the Lorenz's formalism is universal. Therefore we shall substitute in it other, real a velocity of an electromagnetic process. Only two velocities limit the choice: the group U_g and the phase U_f velocity. Because the question is of a moving energy in this case, the choice should be stopped on a group velocity. But it is connected with phase velocity a relation (П2.13). Having taken advantage of this formula, we gain:

$$m_e = \varepsilon_0 \varepsilon \frac{\mathbf{U}e_f{}^2 \mathbf{E}^2}{2c^4}, \quad m_h = \mu_0 \mu \frac{\mathbf{U}h_f{}^2 \mathbf{H}^2}{2c^4}. \qquad (\text{П}2.21)$$

The combination from (П2.20) and (П2.21) gives the formula for inertial fields **E*** and **H***. They can be substituted in required differences of fields $s\mathbf{E} = \mathbf{E} - \mathbf{E}^*$ also $s\mathbf{H} = \mathbf{H} - \mathbf{H}^*$, which are necessary to us for substitution in the dynamic equations of the field. In a view of told we shall write down:

$$s\mathbf{E} = \mathbf{E} - \varepsilon_0 \varepsilon \cdot \boldsymbol{\rho}_e{}^{-1} \frac{\mathbf{U}e_f{}^2 \mathbf{E}^2}{2c^4} \frac{d\mathbf{U}e}{dt} \quad \text{and} \quad s\mathbf{H} = \mathbf{H} - \mu_0 \mu \cdot \boldsymbol{\rho}_h{}^{-1} \frac{\mathbf{U}h_f{}^2 \mathbf{H}^2}{2c^4} \frac{d\mathbf{U}h}{dt}. \qquad (\text{П}2.22)$$

Further differences of fields s**E** and s**H** is substituted in the equations (П2.11) and (П2.12). The result of this operation looks so:

$$\frac{d(s\mathbf{E})}{dt} = \frac{\boldsymbol{\varepsilon}^{-1}}{\varepsilon_0} rot(s\mathbf{H}) + \frac{d\mathbf{U}e}{d(s\mathbf{E})} - (s\mathbf{E}) div \mathbf{U}e - \boldsymbol{\varepsilon}^{-1}(s\mathbf{E}) \frac{d\varepsilon}{dt}, \qquad (\text{П}2.23)$$

$$\frac{d(s\mathbf{H})}{dt} = -\frac{\boldsymbol{\mu}^{-1}}{\mu_0} rot(s\mathbf{E}) + \frac{d\mathbf{U}h}{d(s\mathbf{H})} - (s\mathbf{H}) div \mathbf{U}h - \boldsymbol{\mu}^{-1}(s\mathbf{H}) \frac{d\mu}{dt}. \qquad (\text{П}2.24)$$

And all system is locked by formulas for calculation of current phase velocities:

$$\frac{d}{dt}\mathbf{E} = \frac{\partial}{\partial t}\mathbf{E} + \frac{d\mathbf{E}}{d\mathbf{U}e_f} \quad \text{and} \quad \frac{d}{dt}\mathbf{H} = \frac{\partial}{\partial t}\mathbf{H} + \frac{d\mathbf{H}}{d\mathbf{U}h_f}. \qquad (\text{П}2.25)$$

Thus, we have gained system from 6 vector equations (П2.22) … (П2.25) which contain 6 unknown components s**E**, **E**, **U**e and s**H**, **H**, **U**h. They should describe evolution of nonlinear vortical system, beginning from that state which is set by an initial field configuration.

7. Now we shall consider a condition of the moments balance.

The balance of the moments is actual at the calculations connected with a finite elements method as the moments of sites of medium in each elementary cube have finite quantity.

The usual shape of connection between fields and the moments of forces looks like [23]:

$$\mathbf{M}e = [\mathbf{p}_e, \mathbf{E}], \qquad \mathbf{M}h = [\mathbf{p}_h, \mathbf{H}], \qquad (\Pi 2.26)$$

where $\mathbf{M}e$ and $\mathbf{M}h$ are moments of electrical and magnetic forces, $\mathbf{p}_e$ and $\mathbf{p}_h$ are corresponding distributions of dipole moments of charges in space. But also $\mathbf{p}_e = \mathbf{D}$ и $\mathbf{p}_h = \mathbf{B}$, that it is possible to show on the basis of the analysis of formulas of the previous item or to gain directly from textbooks for the electrical engineer. Thus, we have formulas:

$$\mathbf{M}e = [\mathbf{D}, \mathbf{E}], \qquad \mathbf{M}h = [\mathbf{B}, \mathbf{H}], \qquad (\Pi 2.27)$$

For the moments of inertial forces equality are fair:

$$\mathbf{M}e^* = g_e \cdot \frac{d\boldsymbol{\omega}_e}{dt}, \qquad \mathbf{M}h^* = g_h \cdot \frac{d\boldsymbol{\omega}_h}{dt}, \qquad (\Pi 2.28)$$

where g_e and g_h are the electrical and magnetic moments of inertia of a cube on Fig. П2.1 divided to its volume.

Considering formulas $rot\mathbf{U}e = 2\boldsymbol{\omega}_e$, $rot\mathbf{U}h = 2\boldsymbol{\omega}_h$ and that on conditions of dynamic balance $\mathbf{M}e + \mathbf{M}e^* = 0$ and $\mathbf{M}h + \mathbf{M}h^* = 0$, we results:

$$[\mathbf{D}, \mathbf{E}] = -g_e \cdot \frac{d}{dt}(rot\mathbf{U}e)/2, \qquad [\mathbf{B}, \mathbf{H}] = -g_h \cdot \frac{d}{dt}(rot\mathbf{U}h)/2. \qquad (\Pi 2.29)$$

Gained here formulas define the required moments of forces acting on each elements of space, limited in the parameters of digitization.

8. The differential equations (П2.22)…(П2.25), (П2.29) represent required dynamic equations of an electromagnetic field in nonlinear moving mediums.

As one would expect, they have appeared nonlinear. And nonlinearities there are even two. The first nonlinearity is induced through nonlinear medium together with it ε_{st} and μ_{st} (see Appendix 1). The second nonlinearity is defined only by an equations structure and by a dependence of mass densities on a velocity. It exists and in medium linear, including absolutely free space. This second nonlinearity at ε_{st}=1 and μ_{st}=1 can be named natural. Most likely, it is responsible for formation of fundamental particles. However for full definiteness more the careful analysis of all solutions of these equations is necessary to make in view of feedback and collective interactions of vortexes through a total general field. I.e. most likely, the return to the Dirac idea about the mutual shielding but is perfect on other base. In any case corresponding experiments do not lose urgency.

Let's note the abundantly clear fact: it is not meaningful to speculate with the concept of a wrong (crooked) space at the interpretation of the dynamic field equations (П2.22)…(П2.25), (П2.29). These equations describe not internal changes and not the external strain of equivalent dielectrics and paramagnetics, but reflect how the fields react to a nonlinearity of medium and the amplitude of its disturbance. The transition of a boat on the gliding is an example of a similar phenomenon in the nature. The wave system is transformed during growth of velocity so, that the effective length of the case increases in a transitive mode. However the boat is the same, and the face of the helmsman is all OK. Water has not changed too. Simply waves have appeared, and the water flow nature of boat has changed. Any hallucinations are absolutely inappropriate here; they are not present at the nature. Hence, the support exclusively on a principle of a relativity Galilee not only is quite natural here, but also simply necessary for the execution of all calculations. Theories of Newton, Maxwell, Hertz and this are formulated initially within the limits of this principle.

Told above does not limit in any way a choice of the coordinates system which are distinct from Cartesian. In this case a situation absolutely same, as with habitual Maxwell equations and with Lagrange mechanics [22]. Naturally, all tensor and vector variables will accept already other view after the change of coordinates. And all restrictions of such invention are connected only with its expediency. As here the essence of physical process is in all wave system, as a physical phenomenon, but not in the system of a coordinates. But the coordinate system is a part of a craft of a knowledge, which refers to as the techniques of calculations.